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Bianchi Type-III Cosmic Strings and Domain Walls Cosmological Model in General Relativity

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Abstract Bianchi type-III space time is considered in the presence of cosmic strings and thick domain walls source in the frame work of general relativity. Exact cosmological models using various cases of $\rho = \alpha\lambda$ and $p = \gamma\rho$ are presented. It is observed that the behavior of these models (with cosmic strings and domain walls), based on their physical and kinematical properties, is found to be identical.

Keywords Cosmic strings · Domain walls · Bianchi type-III space time · Relations $\rho = \alpha\lambda$ and $p = \gamma\rho$

1 Introduction

Bianchi type-III cosmological models are the simplest anisotropic universe models playing an important role in understanding essential features of the universe. In this class of models it is possible to accommodate topological defects (cosmic strings, domain walls, monopoles).

At the very early stage of the evolution of the universe, it is generally assumed that during the phase transition the symmetry of the universe is broken spontaneously. It can give rise to topological stable defects such as strings, domain walls and monopoles [7]. Of all these cosmological structures, cosmic strings and domain walls have excited the most interest. The study of cosmic strings and domain walls has received considerable attention in cosmology since they play an important role in structure formation and evolution of the universe [8, 17]. The gravitational effects of cosmic strings have been extensively discussed by Vilenkin [17], Gott [3], Latelier [9], Stachel [16] in general relativity. In particular, the domain walls have become important in recent years from cosmological stand point when a new scenario of galaxy formation has been proposed by Hill et al. [4].

Also Vilenkin [17], Ipser and Sikivie [5], Widrow [19], Goetz [2], Mukherjee [11], Wang [18]), Rahmann and Bera [13], Reddy and Subba Rao [15], Adhav et al. [1] are some authors who have investigated several aspects of domain walls. Recently a lot work has been done

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on strings cosmological models and thick domain walls in Lyra geometry. Mahanta and Mukherjee [10], Rahaman et al. [12] and Reddy [14] have investigated string cosmological models in Lyra geometry. While Reddy and Subba Rao [15] investigated axially symmetric cosmic strings and domain walls in Lyra geometry.

The purpose of present work is to obtain Bianchi type-III cosmological models with cosmic strings and domain walls in general relativity. Our paper is organized as follows.

In Sect. 2, we have derived the field equations of Bianchi type-III cosmic string model.

Section 3 deals with the exact solutions and physical/kinematical properties in different cases of cosmic strings. In Sect. 4, we have derived the field equations of Bianchi type-III models with domain walls. Section 5 is devoted to the exact solutions and physical/kinematical properties in different cases of domain walls. The last section contains conclusion.

2 The Cosmic String Model

We consider the Bianchi type-III metric of the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2ax} dy^2 - C^2 dz^2, \tag{2.1}$$

where a is non zero constant. A, B, C are functions of time 't'.

The energy momentum tensor for a system of cosmic strings in coordinate system is given by

$$T^i_j = \rho u_j u^i - \lambda x_j x^i, \tag{2.2}$$

where ρ is the rest energy density of strings with massive particles attached to them and can be expressed as $\rho = \rho_p + \lambda$, where ρ_p is the rest energy density of the particles attached to the strings and λ is the tension density of the strings.

Here u_i is the four velocity and x_i is the direction of string, obeying the relations

$$u_i u^i = -x_i x^i = 1, \qquad u_i x^i = 0. \tag{2.3}$$

The cosmic string source is along z-axis which is axis of symmetry.

In coordinate system, we have from (2.2)

$$T^1_1 = T^2_2 = 0, \qquad T^3_3 = \lambda, \qquad T^4_4 = \rho, \qquad T^i_j = 0 \quad \text{for } i \neq j \tag{2.4}$$

The quantities ρ and λ depends on t only.

The Einstein's field equations in general relativity are

$$R^j_i - \frac{1}{2} R g^j_i = -8\pi T^j_i \tag{2.5}$$

Now, with the help of (2.2), (2.3) and (2.4), the field equation (2.5) for the metric (2.1) gives

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} = 0 \tag{2.6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C} = 0 \tag{2.7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{a^2}{A^2} = 8\pi\lambda \tag{2.8}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4}{B}\frac{C_4}{C} - \frac{a^2}{A^2} = 8\pi\rho \tag{2.9}$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0. \tag{2.10}$$

From (2.10), we have

$$A = \mu B \tag{2.11}$$

Using (2.11), (2.6) to (2.9) reduce to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} = 0 \tag{2.12}$$

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{a^2}{\mu^2 B^2} = 8\pi\lambda \tag{2.13}$$

$$\left(\frac{B_4}{B}\right)^2 + 2\frac{B_4 C_4}{BC} - \frac{a^2}{\mu^2 B^2} = 8\pi\rho \tag{2.14}$$

3 Cosmic Strings Cases

The field equations (2.12) to (2.14) are only three equations in four unknowns B, C, ρ and λ. Hence to obtain a determinate solution one has to assume one physical or mathematical condition.

In the literature there exists number of relations between ρ and λ, the simplest one being a proportionality relation given by

$$\rho = \alpha\lambda. \tag{3.1}$$

With the most usual choices of the constant α, we get

$$\begin{aligned} \alpha &= 1 \quad \text{(Geometric string or Nambu string)} \\ &= -1 \quad \text{(Reddy string)} \\ &= 1+\omega, \quad \omega \geq 0 \quad \text{(p-string or Takabayasi string).} \end{aligned}$$

Case I: Geometric String ($\rho = \lambda$)

From (2.13) and (2.14), we get

$$\frac{B_{44}}{B} - \frac{B_4 C_4}{BC} = 0 \tag{3.2}$$

Here we also assume the relation $C = B^n$ between the metric coefficients. Using this relation, the field equations (2.12), (2.13) and (2.14) admit the exact solution

$$\begin{aligned} A &= L(k_1 t + k_2)^{\frac{1}{1-n}}, \quad \text{where } L = \mu M \text{ and } n \neq 1 \\ B &= M(k_1 t + k_2)^{\frac{1}{1-n}}, \quad \text{where } M = (1-n)^{\frac{1}{1-n}} \\ C &= N(k_1 t + k_2)^{\frac{n}{1-n}}, \quad \text{where } N = M^n \end{aligned} \tag{3.3}$$

$$8\pi\lambda = \frac{(2n+1)k_1^2}{(1-n)^2(k_1 t + k_2)^2} - \frac{a^2}{L^2(k_1 t + k_2)^{\frac{1}{1-n}}} \tag{3.4}$$

The geometric or Nambu string model in Einstein general theory of relativity corresponding to the above solution can be written as

$$ds^2 = dt^2 - L^2 (k_1 t + k_2)^{\frac{2}{1-n}} dx^2 - M^2 (k_1 t + k_2)^{\frac{2}{1-n}} e^{-2ax} dy^2 - N^2 (k_1 t + k_2)^{\frac{2n}{1-n}} dz^2 \tag{3.5}$$

After a proper choice of coordinates and constants, (3.5) can be written as

$$ds^2 = \frac{dT^2}{k_1^2} - L^2 (T)^{\frac{2}{1-n}} dX^2 - M^2 (T)^{\frac{2}{1-n}} e^{-2aX} dY^2 - N^2 (T)^{\frac{2n}{1-n}} dZ^2 \tag{3.6}$$

The physical parameters for the model (3.6) are

Spatial volume:

$$v = \sqrt{-g} = \frac{LMN}{k_1} (T)^{\frac{1+n}{1-n}} e^{-ax} \tag{3.7}$$

Scalar expansion:

$$\theta = \frac{(2+n)k_1}{(1-n)T} \tag{3.8}$$

Shear scalar:

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{6} \frac{(2+n)^2 k_1^2}{(1-n)^2 T^2} \tag{3.9}$$

Case II: Reddy String ($\rho + \lambda = 0$)

From (2.13) and (2.14), we have

$$\frac{B_{44}}{B} + \left(\frac{B_4}{B} \right)^2 + \frac{B_4 C_4}{BC} = \frac{a^2}{\mu^2 B^2} \tag{3.10}$$

Here again assuming the same relation as in Case I between the metric coefficients, the field equations (2.12), (2.13) and (2.14) admit the exact solution

$$\begin{aligned} A &= L_1 (t + k_2), \quad \text{where } L_1 = \mu M_1 \\ B &= M_1 (t + k_2), \quad \text{where } M_1 = \sqrt{\frac{a^2}{\mu(n+1)}} \\ C &= N_1 (t + k_2)^n, \quad \text{where } N_1 = (M_1)^n \end{aligned} \tag{3.11}$$

$$8\pi\lambda = \left(1 - \frac{a^2}{\mu^2 M_1^2} \right) \frac{1}{T^2} \tag{3.12}$$

Reddy string model corresponding to the solution (3.11) can be written as

$$ds^2 = dt^2 - L_1^2 (t + k_2)^2 dx^2 - M_1^2 (t + k_2)^2 e^{-2ax} dy^2 - N_1^2 (t + k_2)^{2n} dz^2 \tag{3.13}$$

After the proper choice of coordinates and constants, (3.13) can be written as

$$ds^2 = dT^2 - L_1^2 (T)^2 dX^2 - M_1^2 (T)^2 e^{-2aX} dY^2 - N_1^2 (T)^{2n} dZ^2 \tag{3.14}$$

The physical properties for the model (3.14) are

Spatial volume:

$$v = \sqrt{-g} = (L_1 M_1 N_1) T^{n+2} e^{-ax} \tag{3.15}$$

Scalar expansion:

$$\theta = \frac{n+2}{T} \tag{3.16}$$

Shear scalar:

$$\sigma^2 = \sigma_{ij}\sigma^{ij} = \frac{1}{6}\left(\frac{n+2}{T}\right)^2 \tag{3.17}$$

Case III: p-string $[\rho = (1+\omega)\lambda]$

From (2.13) and (2.14), we have

$$2(1+\omega)\frac{B_{44}}{B} + \omega\left(\frac{B_4}{B}\right)^2 - \frac{2B_4C_4}{BC} = \frac{\omega a^2}{\mu^2 B^2} \tag{3.18}$$

Again using the relation $C = B^n$, the field equations (2.12), (2.13) and (2.14) admit the exact solution

$$\begin{aligned}
A &= L_2(t+k)^{\frac{1}{2}}, \quad \text{where } L_2 = \mu M_2 \\
B &= M_2(t+k)^{\frac{1}{2}}, \quad \text{where } M_2 = \frac{\sqrt{2}wa^2}{-\mu^2(2n+2+\omega)} \\
C &= N_2(t+k)^{\frac{2}{n}}, \quad \text{where } N_2 = (M_2)^n
\end{aligned} \tag{3.19}$$

$$8\pi\rho = \left(\frac{n+8}{4n}\right)\frac{1}{T^2} - \frac{a}{\mu^2 M_2^2 T} \tag{3.20}$$

$$8\pi\lambda = \frac{-a^2}{\mu^2 M_2^2 T} \tag{3.21}$$

The p-string model corresponding to the solution (3.19) can be written as

$$ds^2 = dt^2 - L_2^2(t+k)\,dx^2 - M_2^2(t+k)\,e^{-2ax}dy^2 - N_2^2(t+k)^{\frac{4}{n}}\,dz^2 \tag{3.22}$$

After the proper choice of coordinates and constants, (3.22) can be written as

$$ds^2 = dT^2 - L_2^2(T)\,dX^2 - M_2^2(T)\,e^{-2aX}dY^2 - N_2^2(T)^{\frac{4}{n}}\,dZ^2 \tag{3.23}$$

The physical parameters of the model (3.23) are

Spatial volume:

$$v = \sqrt{-g} = (L_2 M_2 N_2) T^{\frac{n+2}{n}} e^{-ax} \tag{3.24}$$

Scalar expansion:

$$\theta = \left(\frac{n+2}{n}\right)\frac{1}{T} \tag{3.25}$$

Shear scalar:

$$\sigma^2 = \frac{1}{6}\left(\frac{n+2}{n}\right)^2\frac{1}{T^2} \tag{3.26}$$

The energy density and tension density of the strings are given by (3.4), (3.12), (3.20) and (3.21). It is observed that at initial moment ($T=0$) the rest energy density ρ and tension density λ diverges.

From (3.7), (3.15) and (3.24), it is observed that at initial moment ($T=0$), the spatial volume will be zero. For large values of T ($T\to\infty$), the spatial volume tends to infinity.

From (3.8), (3.16) and (3.25), it is observed that when $T\to 0$, the expansion scalar θ tends to infinity. While for large value of T ($T\to\infty$), the expansion scalar θ becomes zero.

From (3.9), (3.17) and (3.26), it is observed that at initial moment ($T\to 0$), shear scalar σ^2 tends to infinity. For large values of T ($T\to\infty$), shear scalar σ^2 becomes zero.

Also $\lim_{T\to\infty}(\frac{\sigma}{\theta})^2\neq 0$and hence the model does not approach isotropy.

4 Domain Wall Model and Cases

A thick domain wall can be viewed as soliton-like solution of the scalar field equations coupled with gravity. There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy momentum tensor describing a scalar field ϕ with self interactions contained in a potential $\gamma(\psi)$ given by

$$\psi,_i\psi,_j - g_{ij}\left[\frac{1}{2}\psi,_k\psi^{,k} - \gamma(\psi)\right]$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho\left(g_{ij}+\omega_i\omega_j\right) + p\omega_i\omega_j, \qquad \omega^i\omega_j = -1 \tag{4.1}$$

where ρ is the energy density of the wall, p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space-like vector in the same direction [13].

Here we use the second approach to study the thick domain walls in general relativity. We again consider Bianchi type-III metric given by (2.1).

In the co-moving coordinate system, we have from (4.1)

$$T_4^4 = T_1^1 = T_2^2 = \rho \quad \text{and} \quad T_3^3 = -p, \qquad T_i^j = 0 \quad \text{for } i\neq j, \tag{4.2}$$

(here pressure is taken in the direction of z-axis) the quantities ρ and p depend on t only.

The field equation (2.5) of general relativity with the help of (2.1), (2.11), (4.1) and (4.2) reduce to the following independent equations

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} = 8\pi\rho \tag{4.3}$$

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{a^2}{\mu^2 B^2} = -8\pi p \tag{4.4}$$

$$\left(\frac{B_4}{B}\right)^2 + 2\frac{B_4C_4}{BC} - \frac{a^2}{\mu^2 B^2} = 8\pi\rho \tag{4.5}$$

The field equations (4.3), (4.4) and (4.5) are three equations in four unknowns B, C, ρ and p. Hence to get a determinate solution one has to assume one additional physical or mathematical condition. In the literature, there exists a number of relations between p and ρ, the simplest one being a proportionality relation or equation of state

$$p = \gamma\rho, \tag{4.6}$$

where γ is a constant which lies in the interval [0, 1]. Depending on it's numerical value, γ describes the following types of universes [6]:

$$\begin{aligned}\gamma &= 0 \quad \text{(dust universe)}\\ &= 1/3 \quad \text{(radiation universe)}\\ &= 1 \quad \text{(Zel'dovich universe or stiff matter).}\end{aligned}$$

Case I: Dust Universe ($\gamma = 0$)
Using this case, (4.6) becomes

$$p = 0 \tag{4.7}$$

Using (4.7), (4.3), (4.4) and (4.5), we have

$$3\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{B_4 C_4}{BC} = 0 \tag{4.8}$$

Here we also assume the relation between the metric coefficients as $C = B^n$. Using this relation, the field equations (4.3), (4.4) and (4.5) admit the exact solution

$$\begin{aligned}A &= N_1(k_1 t + k_2)^{\frac{1}{N+1}}, \quad \text{where } N_1 = \mu N_2\\ B &= N_2(k_1 t + k_2)^{\frac{1}{N+1}}, \quad \text{where } N_2 = (N+1)^{\frac{1}{N+1}}\\ C &= N_3(k_1 t + k_2)^{\frac{n}{N+1}}, \quad \text{where } N_3 = (N_2)^n,\end{aligned} \tag{4.9}$$

where N is a constant of integration.

The dust universe model corresponding to the solution (4.9) can be written as

$$ds^2 = dt^2 - N_1^2 (k_1 t + k_2)^{\frac{2}{N+1}} dx^2 - N_2^2 (k_1 t + k_2)^{\frac{2}{N+1}} e^{-2ax} dy^2 - N_3^2 (k_1 t + k_2)^{\frac{2n}{N+1}} dz^2 \tag{4.10}$$

After a proper choice of coordinates and constants, (4.10) reduces to

$$ds^2 = \frac{dT^2}{k_1^2} - N_1^2 (T)^{\frac{2}{N+1}} dX^2 - N_2^2 (T)^{\frac{2}{N+1}} e^{-2aX} dY^2 - N_3^2 (T)^{\frac{2n}{N+1}} dZ^2 \tag{4.11}$$

The physical and kinematical properties for the model (4.11) are

Energy density:

$$8\pi\rho = \frac{k_1^2[n^2 - N(n+1)]}{(N+1)^2 T^2} \tag{4.12}$$

$$8\pi p = 0 \tag{4.13}$$

Spatial volume:

$$v = \sqrt{-g} = \left(\frac{N_1 N_2 N_3}{k_1}\right)(T)^{\frac{n+2}{N+2}} e^{-ax} \tag{4.14}$$

Scalar expansion:

$$\theta = \frac{k_1(n+2)}{(N+1)T} \tag{4.15}$$

Shear scalar:

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{6}\left[\frac{k_1(n+2)}{N+1}\right]^2 \frac{1}{T^2} \tag{4.16}$$

Deceleration parameter:

$$q = +ve \tag{4.17}$$

Case II: Radiation Universe ($\gamma = \frac{1}{3}$)
Using above relation, (4.6) becomes

$$p = \frac{1}{3}\rho \tag{4.18}$$

Using (4.18) with (4.3) to (4.5), we get

$$13\frac{B_{44}}{B} + \frac{C_{44}}{C} + 7\left(\frac{B_4}{B}\right)^2 + 3\frac{B_4 C_4}{BC} - 7\frac{a^2}{\mu^2 B^2} = 0 \tag{4.19}$$

Using the relation $C = B^n$, the field equation (4.19) reduces to

$$\frac{B_{44}}{B} + L\left(\frac{B_4}{B}\right)^2 = \left(\frac{7}{n+13}\right)\frac{a^2}{\mu^2 B^2} \tag{4.20}$$

where $L = \frac{n^2+2n-7}{n+13}$.

Which further leads to

$$BB_{44} + LB_4^2 = \left(\frac{7}{n+13}\right)\frac{a^2}{\mu^2} \tag{4.21}$$

Let us consider

$$B_4 = f(B) \tag{4.22}$$

$$\therefore B_{44} = ff', \quad \text{where } f' = \frac{df}{dB} \tag{4.23}$$

Using above equations (4.21) becomes

$$\frac{d}{dB}(B^{2L}f^2) = \left(\frac{14}{n+13}\right)\frac{a^2}{\mu^2}B^{2L-1} \tag{4.24}$$

After integrating, we get

$$f = \sqrt{\left(\frac{7}{n+13}\right)\frac{a^2}{\mu^2 L} + k_1 B^{-2L}} \tag{4.25}$$

To get determinate solution, we assume that $k_1 = 0$.

From (4.22) and (4.25), we get

$$db = \sqrt{\left(\frac{7}{n+13}\right)\frac{a^2}{\mu^2 L}}dt \tag{4.26}$$

Integrating (4.26), we get

$$B = L_2(t + k_2), \quad \text{where } L_2 = \sqrt{\left(\frac{7}{n+13}\right)\frac{a^2}{\mu^2 L}} \tag{4.27}$$

With the help of (4.27), (2.11) and relation $C = B^n$, we obtain

$$A = L_1(t + k_2), \quad \text{where } L_1 = \mu L_2 \tag{4.28}$$

$$C = L_3(t + k_2)^n, \quad \text{where } L_3 = L_2^n \tag{4.29}$$

The radiation model corresponding to the solutions (4.27) to (4.29) can be written as

$$ds^2 = dt^2 - L_1^2(t + k_2)^2 dx^2 - L_2^2(t + k_2)^2 e^{-2ax} dy^2 - L_3^2(t + k_2)^{2n} dz^2$$

After a proper choice of coordinates and constants, the above equation takes the form

$$ds^2 = dT^2 - L_1^2 T^2 dX^2 - L_2^2 T^2 e^{-2aX} dY^2 - L_3^2 T^{2n} dZ^2 \tag{4.30}$$

The physical and kinematical properties of the model (4.30) are

$$8\pi\rho = \left(3 - \frac{a^2}{\mu^2 L_2^2}\right)\frac{1}{T^2} \quad \text{or} \quad 8\pi\rho = \frac{n}{T^2} \tag{4.31}$$

$$8\pi p = \left(\frac{a^2}{\mu^2 L_2^2} - 1\right)\frac{1}{T^2} \tag{4.32}$$

Spatial volume:

$$v = \sqrt{-g} = (L_1 L_2 L_3) T^{n+2} e^{-ax} \tag{4.33}$$

Scalar expansion:

$$\theta = \frac{n+2}{T} \tag{4.34}$$

Shear scalar:

$$\sigma^2 = \frac{1}{6}\left(\frac{n+2}{T}\right)^2 \tag{4.35}$$

Deceleration parameter:

$$q = -\left[-\frac{3}{n+2} + 1\right] = +ve \tag{4.36}$$

Case III: Zel'dovich Universe or Stiff Matter ($\gamma = 1$)
Using this case, (4.6) becomes

$$p = \rho \tag{4.37}$$

Using (4.3), (4.4), (4.5) and (4.37), we have

$$5\frac{B_{44}}{B} + \frac{C_{44}}{C} + 3\left(\frac{B_4}{B}\right)^2 + 3\frac{B_4 C_4}{BC} - 3\frac{a^2}{\mu^2 B^2} = 0 \tag{4.38}$$

Again assuming the relation between metric coefficients as $C = B^n$, the field equations (4.3) to (4.5) admit the exact solution

$$\begin{aligned} A &= M_1(t+k_2), \quad \text{where } M_1 = \mu M_2 \\ B &= M_2(t+k_2), \quad \text{where } M_2 = \sqrt{\left(\frac{6}{n+5}\right)\frac{a^2}{\mu^2 2M}} \\ C &= M_3(t+k_2)^n, \quad \text{where } M_3 = (M_2)^n \end{aligned} \tag{4.39}$$

The Zel'dovich model corresponding to the solution (4.39) can be written as

$$ds^2 = dt^2 - M_1^2\,(t+k_2)^2\,dx^2 - M_2^2\,(t+k_2)^2\,e^{-2ax}\,dy^2 - M_3^2\,(t+k_2)^{2n}\,dz^2$$

After a proper choice of coordinates and constants, the above equation takes the form

$$ds^2 = dT^2 - M_1^2 T^2 dX^2 - M_2^2 T^2 e^{-2aX} dY^2 - M_3^2 T^{2n} dZ^2 \tag{4.40}$$

The physical and kinematical properties for the model (4.40) are

$$8\pi\rho = 8\pi p = (\text{constant})\frac{1}{T^2} \tag{4.41}$$

Spatial volume:

$$v = \sqrt{-g} = (M_1 M_2 M_3) T^{n+2} e^{-ax} \tag{4.42}$$

Scalar expansion:

$$\theta = \frac{n+2}{T} \tag{4.43}$$

Shear scalar:

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{6}\left(\frac{n+2}{T}\right)^2 \tag{4.44}$$

Deceleration parameter:

$$q = +ve \tag{4.45}$$

The energy density and pressure of the domain walls are given by (4.12), (4.13), (4.31), (4.32) and (4.41) respectively.

From (4.14), (4.33) and (4.42), it is observed that at initial moment ($T = 0$), the spatial volume will be zero. For large values of T ($T \to \infty$), the spatial volume tends to infinity.

From (4.15), (4.34) and (4.43), it is observed that when $T \to 0$, the expansion scalar θ tends to infinity. While for large value of T ($T \to \infty$), the expansion scalar θ becomes zero.

From (4.16), (4.35) and (4.44), it is observed that at initial moment ($T \to 0$), shear scalar σ^2 tends to infinity. For large values of T ($T \to \infty$), shear scalar σ^2 becomes zero.

From (4.17), (4.36) and (4.45), the positive value of the deceleration parameter q indicates that the model decelerates in a standard way.

Also $\lim_{T\to\infty}(\frac{\sigma}{\theta})^2 \neq 0$ and hence the model does not approach isotropy.

5 Conclusion

Cosmic strings, thick domain walls and space times associated with them have cosmological interest due to their important applications in structure formation of the universe. Here we have presented non-static Bianchi type-III cosmological models with cosmic strings and thick domain walls in Einstein's general theory of relativity. The models in all the cases behave alike. The cosmological models studied here will be useful for better understanding of structure formation of the universe.

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